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# The holographic Weyl anomaly

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Abstract: We calculate the Weyl anomaly for conformal field theories that can be described via the $adS$/CFT correspondence. This entails regularizing the gravitational part of the corresponding supergravity action in a manner consistent with general covariance. Up to a constant, the anomaly only depends on the dimension $d$ of the manifold on which the conformal field theory is defined. We present concrete expressions for the anomaly in the physically relevant cases $d = 2, 4$ and 6. In $d = 2$ we find for the central charge $c = 3l/2G_N$, in agreement with considerations based on the asymptotic symmetry algebra of $adS_3$. In $d = 4$ the anomaly agrees precisely with that of the corresponding $\mathcal{N} = 4$ superconformal $SU(N)$ gauge theory. The result in $d = 6$ provides new information for the $(0, 2)$ theory, since its Weyl anomaly has not been computed previously. The anomaly in this case grows as $N^3$, where $N$ is the number of coincident $M5$ branes, and it vanishes for a Ricci-flat background.

Keywords: 1/N Expansion, M-Theory, Brane Dynamics in Gauge Theories, Anomalies in Field and String Theories.

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## Contents





## 1. Introduction

At low energies, the worldvolume theory on $N$ coincident $p$-branes in $M$-theory or string theory decouples from the bulk theory and can be studied on its own. In some cases, the worldvolume theory constitutes a conformal field theory (CFT). This is true, for example, for $D3$-branes in type $IIB$ string theory and for five-branes in $M$-theory, which give rise to the $d = 4$ $\mathcal{N} = 4$ superconformal $SU(N)$ gauge theory and a $d = 6$ $(0, 2)$ superconformal field theory, respectively. It has recently been conjectured by Maldacena [1], following earlier work on black holes [2, 3, 4, 5, 6], that these conformal field theories are dual to $M$-theory or string theory in the background describing the near-horizon brane configuration. This equivalence may also be inferred by observing that the brane configuration can be mapped to its near-horizon limit [7] by means of certain duality transformations [8]. However, this argument is as yet incomplete, since these duality transformations are not fully understood, as they involve the time coordinate. The correspondence between string theory in a specific background and conformal field theories is a realization of the holographic principle advocated by 't Hooft [9] and Susskind [10] in that it describes a $(d + 1)$-dimensional theory containing gravity in terms of degrees of freedom on a $d$-dimensional hypersurface.

The conjectured correspondence was clarified by Gubser, Klebanov and Polyakov [11] and by Witten [12] as follows: the supergravity background is a product of a compact manifold and a $(d+1)$-dimensional manifold $X_{d+1}$ with a boundary ("horizon") $M_d$. The conformal field theory is defined on $M_d$. There is a one-to-one relationship between operators $\mathcal{O}$ of the conformal field theory and the fields $\phi$ of the supergravity theory. In particular, gauge fields in the bulk couple to global currents in the boundary. The presence of a boundary means that the supergravity action functional $S[\phi]$ must be supplemented by a boundary condition for $\phi$ parametrized by a field $\phi^{(0)}$ on $M_d$. The partition function is then a functional of the boundary conditions

$$Z_{string}[\phi^{(0)}] = \int_{\phi^{(0)}} \mathcal{D}\phi \exp\left(-S[\phi]\right), \tag{1.1}$$

where the subscript $\phi^{(0)}$ on the integral sign indicates that the functional integral is over field configurations $\phi$ that satisfy the boundary condition given by $\phi^{(0)}$. The conjecture states that the string (or $M$-theory) partition function, as a functional of $\phi^{(0)}$, equals the generating functional of correlation functions in the conformal field theory:

$$Z_{CFT}[\phi^{(0)}] = \left\langle \exp \int_{M_d} d^d x \mathcal{O}\phi^{(0)} \right\rangle . \tag{1.2}$$

The fields $\phi^{(0)}$ act as sources for the operators of the conformal field theory. Notice that the bulk theory only sees, through the boundary values of its fields, the abstract conformal field theory and not the elementary fields that may realize it. The partition function (1.2) may also be viewed as describing the coupling of conformal matter to conformal supergravity [13]. The sources $\phi^{(0)}$ constitute conformal supermultiplets.

The relationship just described is conjectured to hold for any number $N$ of coincident branes. However, in most cases one can reliably compute the string partition function only for large $N$. The reason is that the backgrounds involve RR forms whose coupling to perturbative strings is through $D$-branes. Therefore a complete string calculation is rather difficult to perform. However, if the number of branes is large, the characteristic length scale of the supergravity background is large compared to the string scale (or the Planck scale in the case of $M$-theory), and one can trust the supergravity approximation. In addition the string coupling may be chosen small. Under these circumstances, the string partition function reduces to the exponential of the supergravity action functional evaluated for a field configuration $\phi^{cl}(\phi^{(0)})$ that solves the classical equations of motion and satisfies the boundary conditions given by $\phi^{(0)}$

$$Z^{tree}_{string}[\phi^{(0)}] = \exp\left(-S[\phi^{cl}(\phi^{(0)})]\right). \tag{1.3}$$

An operator of particular importance in any conformal field theory is the energy-momentum tensor $T_{ij}$, $i,j=1,\ldots,d$. The corresponding bulk gauge field is the metric $\hat{G}_{\mu\nu}$, $\mu,\nu=0,\ldots,d$ on $X_{d+1}$. In the supergravity backgrounds under consideration, the metric $\hat{G}_{\mu\nu}$ does not induce a unique metric $g_{(0)}$ on the boundary $M_d$, because it has a second-order pole there. However it does determine a conformal equivalence class or conformal structure $[g_{(0)}]$ of metrics on $M_d$. To get a representative $g_{(0)}$, we pick a function $\rho$ on $X_{d+1}$ with a simple zero on $M_d$ and restrict $\rho^2\hat{G}_{\mu\nu}$ to $M_d$. Different choices of the function $\rho$ yield different metrics on $M_d$ in the same conformal equivalence class. The field that specifies the boundary condition of the metric is thus a conformal structure $[g_{(0)}]$. This means that, at least naively, the trace of the energy-momentum tensor decouples.

In this paper we wish to determine the dependence of the boundary theory partition function (or zero-point function) on a given representative $g_{(0)}$ of the conformal structure. In other words, we shall study whether the trace of the energy-momentum decouples. Since we examine correlation functions that only involve the energy-momentum tensor, the only relevant part of the bulk action is the gravitational one. Therefore we set all other fields to zero. At tree level, we then need to solve the classical supergravity equations of motion on $X_{d+1}$ subject to the conditions that the metric $\hat{G}_{\mu\nu}$ on $X_{d+1}$ induces a given conformal structure $[g_{(0)}]$ on $M_d$ and all other fields vanish there. In the theories under consideration, this means that $\hat{G}_{\mu\nu}$ fulfills Einstein's equations

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{G}_{\mu\nu}\hat{R} = \Lambda \hat{G}_{\mu\nu}\,, \tag{1.4}$$

with some cosmological constant $\Lambda$ and that all other fields vanish identically on $X_{d+1}$. According to a theorem due to Graham and Lee [14], up to diffeomorphism, there is a unique such metric $\hat{G}_{\mu\nu}$. (Actually the theorem has been proved for the case when $X_{d+1}$ is topologically a ball $B_{d+1}$ so that $M_d$ is a sphere $S^d$ and the conformal structure $[g_{(0)}]$ on $M_d$ is sufficiently close to the standard (conformally flat) one.) The conformal field theory effective action (strictly speaking, the generating functional of the connected graphs) $W_{CFT}[g_{(0)}] = -\log Z_{CFT}[g_{(0)}]$ is then given by evaluating the action functional

$$\begin{aligned} S[\hat{G}_{\mu\nu}] &= S_{bulk} + S_{boundary} \\ &= \frac{1}{16\pi G_N^{(d+1)}}\left[\int_{X_{d+1}} d^{d+1}x\sqrt{\det \hat{G}}\left(\hat{R}+2\Lambda\right) + \int_{M_d} d^d x \sqrt{\det \tilde{g}}\,(2D_\mu n^\mu + \alpha)\right] \end{aligned} \tag{1.5}$$

for this metric. Here $\tilde{g}$ is the metric induced on $M_d$ from $\hat{G}$, and $n^\mu$ is a unit normal vector to $M_d$. The bulk term is of course the usual Einstein-Hilbert action with a cosmological constant. The inclusion of the first boundary term is necessary on a manifold with boundary in order to get an action that depends only on first derivatives of the metric [15]. The possibility of including the second boundary term with some coefficient $\alpha$ was first discussed in [13].

The above description might seem to indicate that the conformal field theory effective action $W_{CFT}[g_{(0)}]$ only depends on the conformal equivalence class of the metric on $M_d$. This is of course as it should be in a truly conformally invariant theory. However, the action functional (1.5) does not make sense for the metric $\hat{G}_{\mu\nu}$ determined by (1.4) and the boundary conditions. Indeed, the bulk term of the action diverges because of the infinite volume of $X_{d+1}$. The boundary terms are also ill-defined, since the induced metric $\tilde{g}_{ij}$ on $M_d$ diverges because of the double pole of $\hat{G}_{\mu\nu}$. The action should therefore be regularized in a way that preserves general covariance, so that the divergences can be cancelled by the addition of local counterterms. As we will see shortly, this regularization entails picking a particular, but arbitrary, representative $g_{(0)}$ of the conformal structure $[g_{(0)}]$ on $M_d$. In this way, one obtains a finite effective action, which, however, will depend on the choice of this representative metric. Conformal invariance is thus explicitly broken by a so-called conformal or Weyl anomaly. The anomaly, which is usually perceived as a UV effect, thus arises from an IR-divergence in the bulk theory. This is an example of a more general IR-UV connection that applies to holographic theories [16].

In this paper, we will calculate the Weyl anomaly for conformal field theories that can be derived from a supergravity theory, as described above. In the next section, we will describe the regularization procedure and the computation of the anomaly in general. In the last section, we evaluate the anomaly in the physically relevant cases $d = 2, 4, 6$. For $d = 2$ and $d = 4$ we compare with the known anomaly for the $adS_3$ boundary conformal field theory and the $d = 4$ $\mathcal{N} = 4$ superconformal $SU(N)$ gauge theory respectively, and find perfect agreement. For $d = 6$ there is no corresponding calculation of the Weyl anomaly, so our result provides new information about the $(0, 2)$ superconformal field theory.

## 2. The regularization procedure

A regularization scheme that preserves general covariance was described in [12]. As discussed above, up to diffeomorphisms, there is a unique Einstein metric $\hat{G}$ on $X_{d+1}$ that induces a given a conformal structure $[g_{(0)}]$ on the boundary $M_d$. We now pick a metric $g_{(0)}$ on $M_d$ in the given conformal equivalence class. According to a theorem due to Fefferman and Graham [17], there is a distinguished coordinate system $(\rho, x^i)$ on $X_{d+1}$ in which $\hat{G}$ takes the form

$$\hat{G}_{\mu\nu} dx^\mu dx^\nu = \frac{l^2}{4} \rho^{-2} d\rho d\rho + \rho^{-1} g_{ij} dx^i dx^j \,, \tag{2.1}$$

where the tensor $g$ has the limit $g_{(0)}$ as one approaches the boundary represented by $\rho = 0$. The length scale $l$ is related to the cosmological constant $\Lambda$ as $\Lambda = -\frac{d(d-1)}{2l^2}$. Einstein's equations for $\hat{G}$ amount to

$$\begin{aligned} 0 &= \rho \left(2g'' - 2g'g^{-1}g' + \mathrm{Tr}(g^{-1}g')g'\right) + l^2 \mathrm{Ric}(g) - (d-2)g' - \mathrm{Tr}(g^{-1}g')g \\ 0 &= (g^{-1})^{jk} \left(\nabla_i g'_{jk} - \nabla_k g'_{ij}\right) \\ 0 &= \mathrm{Tr}(g^{-1}g'') - \frac{1}{2}\mathrm{Tr}(g^{-1}g'g^{-1}g') \,, \end{aligned} \tag{2.2}$$

where differentiation with respect to $\rho$ is denoted with a prime, $\nabla_i$ is the covariant derivative constructed from the metric $g$ and $\mathrm{Ric}(g)$ is the Ricci tensor[1] of $g$.

In the case when $d$ is odd, these equations can be solved order by order in $\rho$ so that

$$g = g_{(0)} + \rho g_{(2)} + \rho^2 g_{(4)} + \ldots \,, \tag{2.3}$$

where the tensor $g_{(k)}$ is given by some covariant expression in the boundary metric $g_{(0)}$, its Riemann tensor and the corresponding covariant derivative. Throughout this

[1]Our conventions are as follows $R_{ijk}{}^l = \partial_i \Gamma_{jk}{}^l + \Gamma_{ip}{}^l \Gamma_{jk}{}^p - i \leftrightarrow j$ and $R_{ij} = R_{ikj}{}^k$.



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paper, a subscript in parentheses on a quantity indicates the number of derivatives with respect to $x^i$. In the case when $d$ is even, this procedure breaks down at order $d/2$ in $\rho$, where a logarithmic term appears:

$$g = g_{(0)} + \rho g_{(2)} + \ldots + \rho^{d/2} g_{(d)} + \rho^{d/2} \log \rho \; h_{(d)} + \mathcal{O}(\rho^{d/2+1}) \,. \tag{2.4}$$

The tensors $g_{(k)}$ for $k = 0, 2, \ldots, d-2$ are again covariant. The same is true for $\mathrm{Tr}(g_{(0)}^{-1} g_{(d)})$ but not for the complete tensor $g_{(d)}$. Finally, $\mathrm{Tr}(g_{(0)}^{-1} h_{(d)})$ vanishes identically.

The regularization procedure now amounts to restricting the bulk integral to the domain $\rho > \epsilon$ for some cutoff $\epsilon > 0$ and evaluating the boundary integrals at $\rho = \epsilon$. The regulated action evaluated for the metric $\hat{G}$ is thus $(16\pi G_N^{(d+1)})^{-1} \int d^d x \, \mathcal{L}$, where

$$\mathcal{L} = \frac{d}{l} \int_\epsilon d\rho \, \rho^{-d/2-1} \sqrt{\det g} + \rho^{-d/2} \left( -\frac{2d}{l} \sqrt{\det g} + \frac{4}{l} \rho \partial_\rho \sqrt{\det g} + \alpha \sqrt{\det g} \right) \Bigg|_{\rho=\epsilon} \tag{2.5}$$

In the first term, which arises from the bulk part of the action, we have used the fact that $\hat{G}$ is an Einstein metric so that $\hat{R} + 2\Lambda = -\frac{4}{d-1}\Lambda = \frac{2d}{l^2}$.

For $d$ odd, it follows from (2.3) that $\sqrt{\det g}$ is a power series in $\rho$ with covariant coefficients. For $d$ even, this is true up to and including the $\rho^{d/2}$ terms. (The higher-order non-covariant corrections will play no role in the sequel). The Lagrangian (2.5) can therefore be written as

$$\mathcal{L} = \sqrt{\det g_{(0)}} \left( \epsilon^{-d/2} a_{(0)} + \epsilon^{-d/2+1} a_{(2)} + \ldots + \epsilon^{-1/2} a_{(d-1)} \right) + \mathcal{L}_{fin} \tag{2.6}$$

for $d$ odd, and as

$$\mathcal{L} = \sqrt{\det g_{(0)}} \left( \epsilon^{-d/2} a_{(0)} + \epsilon^{-d/2+1} a_{(2)} + \ldots + \epsilon^{-1} a_{(d-2)} - \log \epsilon \, a_{(d)} \right) + \mathcal{L}_{fin} \tag{2.7}$$

for $d$ even, where $\mathcal{L}_{fin}$ is finite in the $\epsilon \to 0$ limit. All the $a_{(k)}$ coefficients are covariant, so the divergent terms can be cancelled by subtracting covariant counterterms, as promised. The logarithmic divergence that appears for $d$ even comes only from the bulk integral.

After subtraction of the divergent counterterms, we are left with a renormalized effective action $(16\pi G_N^{(d+1)})^{-1} \int d^d x \, \mathcal{L}_{fin}$ with a finite limit as $\epsilon$ goes to zero. Its variation under a conformal transformation $\delta g_{(0)} = 2\delta\sigma g_{(0)}$ for an infinitesimal parameter function $\delta\sigma$ is of the form

$$\delta \mathcal{L}_{fin} = - \int_{M_d} d^d x \sqrt{\det g_{(0)}} \delta\sigma \mathcal{A} \tag{2.8}$$

and we would like to calculate the anomaly $\mathcal{A}$. For $d$ odd, $\mathcal{A}$ in fact vanishes, whereas for $d$ even

$$\mathcal{A} = \frac{1}{16\pi G_N^{(d+1)}} (-2 a_{(d)}) \,. \tag{2.9}$$

To see this, we note that for a *constant* parameter $\delta\sigma$, the regulated Lagrangian (2.6) or (2.7) is invariant under the combined transformation $\delta g_{(0)} = 2\delta\sigma g_{(0)}$ and $\delta\epsilon = 2\delta\sigma\epsilon$.

The terms proportional to negative powers of $\epsilon$ are separately invariant, so the variation of the finite part plus the variation of the logarithmically divergent term (for $d$ even) must vanish. Since $\log\epsilon$ transforms with a shift and $\sqrt{\det g_{(0)}}a_{(d)}$ itself is invariant, we get (2.9).

On general grounds [18, 19], the coefficient $a_{(d)}$ that appears in the anomaly (2.9) must be of the form

$$a_{(d)} = dl^{d-1}\left(E_{(d)} + I_{(d)} + D_i J^i_{(d-1)}\right), \tag{2.10}$$

where $E_{(d)}$ is proportional to the $d$-dimensional Euler density and $I_{(d)}$ is a conformal invariant. These terms are referred to as the type A and the type B anomaly, respectively, in [19]. The dimension of the space of conformal invariants grows with $d$. The $D_i J^i_{(d-1)}$ term, where $D_i$ is the covariant derivative constructed from the boundary metric $g_{(0)}$, is trivial in the sense that it can be cancelled by the variation of a finite covariant counterterm added to the action. To see this, notice that a covariant counterterm will be, in particular, scale invariant. Making the parameter of the scale transformation local amounts to computing the Noether current for scale transformations. Thus, the result of the variation is $\delta\sigma D_i J^i$. However, local scale transformations are just Weyl transformations. Thus terms of the form $D_i J^i$ can be obtained by variation of covariant counterterms.

The coefficients of the various independent contributions (properly normalized) in (2.10) are closely related to renormalization group equations, and they reflect the matter content of the superconformal theory. Using Ward identities one can relate them to Schwinger terms in the OPEs of the energy-momentum tensor [20]–[23]. For a recent application, see [5]. Our results for $d = 6$ can be similarly used to determine Schwinger terms in the OPEs of the $(0, 2)$ theory.

## 3. Evaluation of the anomaly

In this section, we will perform the above procedure in the physically relevant cases $d = 2, 4, 6$ and give concrete formulas for the quantities $E$, $I$ and $J^i$ appearing in (2.10). As we have mentioned in the previous paragraph, the logarithmic divergence comes only from the bulk integral. It is completely straightforward to obtain $a_{(d)}$. One only needs to expand $\sqrt{\det g}$ up to appropriate order in $\rho$. In the formulae below, we further simplify the result by eliminating $\mathrm{Tr}(g_{(0)}^{-1}g_{(n)})$, for $n > 2$, by using the third equation in (2.2). We raise and lower indices with the boundary metric $g_{(0)}$ and its inverse $g_{(0)}^{-1}$. The Riemann tensor and the covariant derivative constructed from $g_{(0)}$ are denoted $R^i{}_{jkl}$ and $D_i$, respectively.

### 3.1. $d = 2$ and the asymptotic symmetry algebra of $adS_3$

Calculating

$$a_{(2)} = l\,\mathrm{Tr}(g_{(0)}^{-1}g_{(2)}) \tag{3.1}$$

and decomposing it according to (2.10), we get

$$\begin{aligned} E_{(2)} &= \frac{1}{4}R \\ I_{(2)} &= 0 \\ J^i_{(1)} &= 0\,. \end{aligned} \tag{3.2}$$

(There is in fact no non-trivial conformal invariant $I$ in this dimension.) Writing the anomaly in the form

$$\mathcal{A} = -\frac{c}{24\pi}R\,, \tag{3.3}$$

(in our conventions a free boson contributes to the anomaly $-1/24\pi R$) we thus get

$$c = \frac{3l}{2G_N^{(3)}}\,. \tag{3.4}$$

This agrees with the value of the conformal anomaly $c$ as computed in [24] by considering the asymptotic symmetry algebra of $adS_3$.

### 3.2. $d = 4$ and $\mathcal{N} = 4$ super Yang-Mills theory

In this case one finds

$$\begin{aligned} a_{(4)} &= l^3\frac{1}{2}\left([\mathrm{Tr}(g^{-1}_{(0)}g_{(2)})]^2 - \mathrm{Tr}[(g^{-1}_{(0)}g_{(2)})^2]\right) \\ &= l^3\left(-\frac{1}{8}R^{ij}R_{ij} + \frac{1}{24}R^2\right). \end{aligned} \tag{3.5}$$

Notice that this expression vanishes for a Ricci-flat background. A check on our calculation is whether (3.5) can be rewritten in the form (2.10). Indeed, this is possible, and we obtain

$$\begin{aligned} E_{(4)} &= \frac{1}{64}\left(R^{ijkl}R_{ijkl} - 4R^{ij}R_{ij} + R^2\right) \\ I_{(4)} &= -\frac{1}{64}\left(R^{ijkl}R_{ijkl} - 2R^{ij}R_{ij} + \frac{1}{3}R^2\right) \\ J^i_{(3)} &= 0\,. \end{aligned} \tag{3.6}$$

(Up to a constant, $I_{(4)}$ is in fact the unique conformal invariant with four derivatives in this dimension, namely the Weyl tensor contracted with itself.) We now use the fact that $G_N^{(5)} = G_N^{(10)}/Vol(S^5)$, where $Vol(S^5) = l^5\pi^3$ is the volume of the compactification five-sphere of radius $l$, and $G_N^{(10)} = 8\pi^6 g_{\mathrm{str}}^2$ is the ten-dimensional Newton's constant (the $\alpha'$'s cancel out in the Maldacena limit). Furthermore, $l$ is related to the number $N$ of $D3$-branes as $l = (4\pi g_{str}N)^{1/4}$ [1]. Putting everything together, we get

$$\mathcal{A} = -\frac{N^2}{\pi^2}\left(E_{(4)} + I_{(4)}\right). \tag{3.7}$$

This should be compared with the conformal anomaly of the $d = 4$ $\mathcal{N} = 4$ superconformal $SU(N)$ gauge theory. The conformal anomaly of a theory with $n_s$ scalar fields, $n_f$ Dirac fermions and $n_v$ vector fields is [25]

$$-\frac{1}{90\pi^2}(n_s + 11n_f + 62n_v)E_{(4)} - \frac{1}{30\pi^2}(n_s + 6n_f + 12n_v)I_{(4)}\,. \tag{3.8}$$

The anomaly of the $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills multiplet is equal to $N^2 - 1$ (as all fields are in the adjoint) times (3.8) for $n_s = 6$, $n_f = 2$ and $n_v = 1$. Thus, in the large $N$ limit we obtain exact agreement with (3.7). This is perhaps surprising since the result (3.8) is derived using free fields whereas our result is about the full interacting $\mathcal{N} = 4$ $SU(N)$ superconformal field theory. This indicates that there must be a non-renormalization theorem that protects these coefficients.

Various orbifolding procedures [26, 27] change the volume of the compactification space and also give rise to other gauge groups. It is easy to check that the anomalies still work out correctly.

### 3.3. $d = 6$ and tensionless strings

Following [18], we introduce

$$\begin{aligned}(K_1, \ldots, K_{11}) = \Big(&R^3, RR_{ij}R^{ij}, RR_{ijkl}R^{ijkl}, R_i{}^jR_j{}^kR_k{}^i, R^{ij}R^{kl}R_{iklj},\\ &R_{ij}R^{iklm}R^j{}_{klm}, R_{ijkl}R^{ijmn}R^{kl}{}_{mn}, R_{ijkl}R^{imnl}R^j{}_{mn}{}^k,\\ &R\Box R, R_{ij}\Box R^{ij}, R_{ijkl}\Box R^{ijkl}\Big)\,.\end{aligned} \tag{3.9}$$

The six-dimensional Euler density is then proportional to

$$E_0 = K_1 - 12K_2 + 3K_3 + 16K_4 - 24K_5 - 24K_6 + 4K_7 + 8K_8 \tag{3.10}$$

and

$$\begin{aligned}I_1 &= \frac{19}{800}K_1 - \frac{57}{160}K_2 + \frac{3}{40}K_3 + \frac{7}{16}K_4 - \frac{9}{8}K_5 - \frac{3}{4}K_6 + K_8\\ I_2 &= \frac{9}{200}K_1 - \frac{27}{40}K_2 + \frac{3}{10}K_3 + \frac{5}{4}K_4 - \frac{3}{2}K_5 - 3K_6 + K_7\\ I_3 &= K_1 - 8K_2 - 2K_3 + 10K_4 - 10K_5 - \frac{1}{2}K_9 + 5K_{10} - 5K_{11}\end{aligned} \tag{3.11}$$

form a basis for conformal invariants with six derivatives.

We have

$$\begin{aligned}a_{(6)} = l^5 \Big(&\frac{1}{8}[\mathrm{Tr} g_{(0)}^{-1}g_{(2)}]^3 - \frac{3}{8}\mathrm{Tr}[g_{(0)}^{-1}g_{(2)}]\mathrm{Tr}[(g_{(0)}^{-1}g_{(2)})^2] +\\ &+\frac{1}{2}\mathrm{Tr}[(g_{(0)}^{-1}g_{(2)})^3] - \mathrm{Tr}[g_{(0)}^{-1}g_{(2)}g_{(0)}^{-1}g_{(4)}]\Big)\,.\end{aligned} \tag{3.12}$$

Evaluating this expression we obtain

$$\begin{aligned}a_{(6)} = \frac{l^5}{64}\Big(&-\frac{1}{2}RR^{ij}R_{ij} + \frac{3}{50}R^3 + R^{ij}R^{kl}R_{ikjl} +\\ &+\frac{1}{5}R^{ij}D_iD_jR - \frac{1}{2}R^{ij}\Box R_{ij} + \frac{1}{20}R\Box R\Big)\,.\end{aligned} \tag{3.13}$$

Observe that the above expression vanishes in a Ricci-flat background. The next task is to put this expression in the form (2.10). This is a nice check on our calculation. The result is

$$
\begin{aligned}
E_{(6)} &= \frac{1}{6912} E_0 \\
I_{(6)} &= \frac{1}{1152}\left(-\frac{10}{3} I_1 - \frac{1}{6} I_2 + \frac{1}{10} I_3\right) \\
J^i_{(5)} &= -\frac{1}{1152}[-R^{ijkl} D^m R_{mjkl} + 2(R_{jk} D^i R^{jk} - R_{jk} D^j R^{ki})] + \\
&\quad + \frac{1}{720} R^{ij} D_j R + \frac{17}{11520} R D^i R \,.
\end{aligned} \tag{3.14}
$$

We now use the fact that $G_N^{(7)} = G_N^{(11)}/Vol(S^4)$, where $Vol(S^4) = R^4_{sph}(8\pi^2/3)$ and $R_{sph} = l_{Planck}(\pi N)^{1/3}$ is the radius of the compactification sphere. In addition, the eleven-dimensional Newton's constant is equal to $G_N^{(11)} = 16\pi^7 l^9_{Planck}$, and the characteristic length $l$ is $l = 2l_{Planck}(\pi N)^{1/3}$. Putting everything together we get for the anomaly

$$
\mathcal{A} = -\frac{4N^3}{\pi^3}\left(E_{(6)} + I_{(6)} + D_i J^i_{(5)}\right) . \tag{3.15}
$$

The anomaly for a $(0,2)$ tensor multiplet has not yet been calculated. However, we see that the anomaly grows as $N^3$, in agreement with considerations based on the entropy of the brane system [28, 5]. This growth is presumably related to the appearance of tensionless strings when multiple fivebranes coincide.

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